Lecture 13
CSMA/CD and Multi-access Reservations

Prof. Krishna R. Pattipati

Dept. of Electrical and Computer Engineering
University of Connecticut
Contact: krishna@engr.uconn.edu (860) 486-2890

EE 336
Stochastic Models for the Analysis of Computer Systems and Communication Networks

Outline of Lecture 12

- Carrier Sense Multipler Access (CSMA)
- Stabilization of CSMA
- CSMA/CD (Collision Detection)
- Multi-access Reservations

CSMA - 1

- CSMA random access

• These are refinements on the pure and Slotted Aloha – We use additional hardware to detect (i.e., sense) the transmissions of other nodes

• Very useful for systems with
propagation delays << packet transmission times

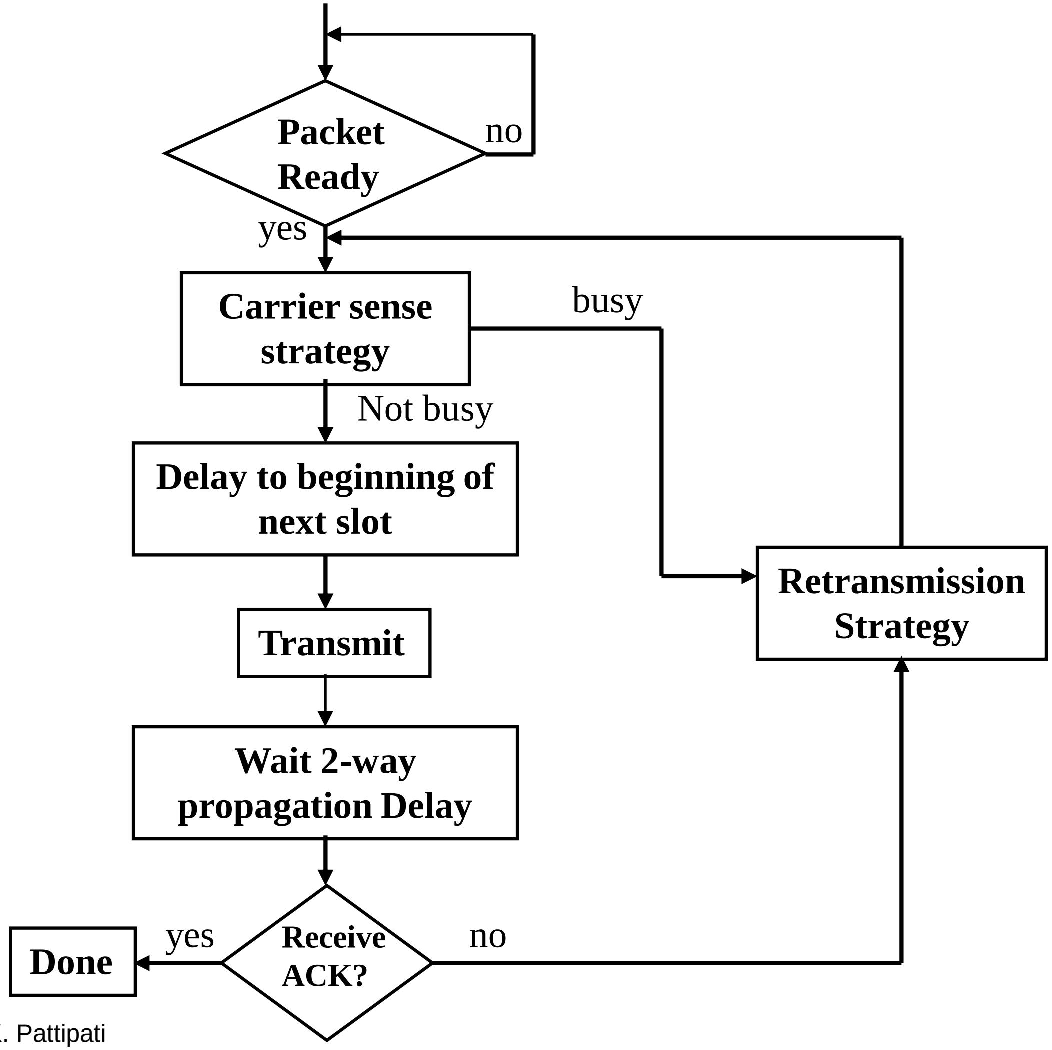
• Can have slotted or unslotted versions.

$Let\ \tau = \text{propagation and detection delay to detect an idle channel}$
$\text{after a transmission ends}$

- $CSMA$ uses τ as the slot size
- If slotted, must tranmit it at the beginning of a slot.

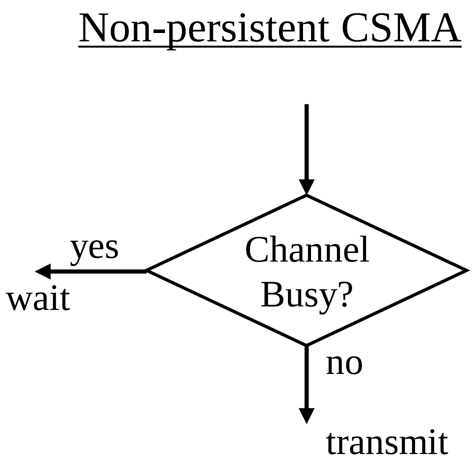
CSMA - 2

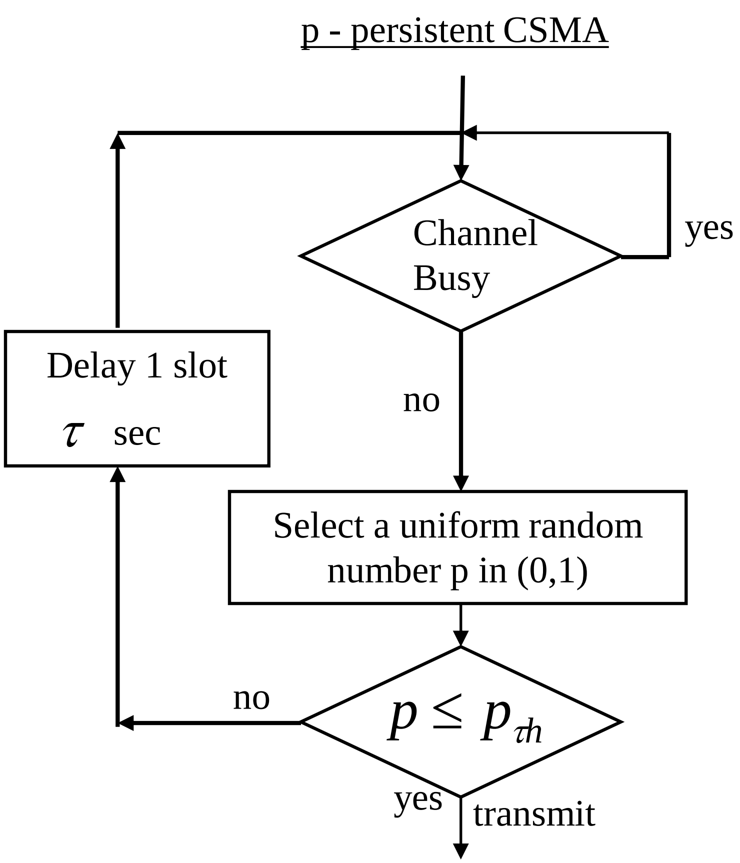
- CSMA random access:



Types of CSMA

- Two types of CSMA:
  - Non-Persistent CSMA
  - p- persistent CSMA





Analysis of Unslotted CSMA - 1

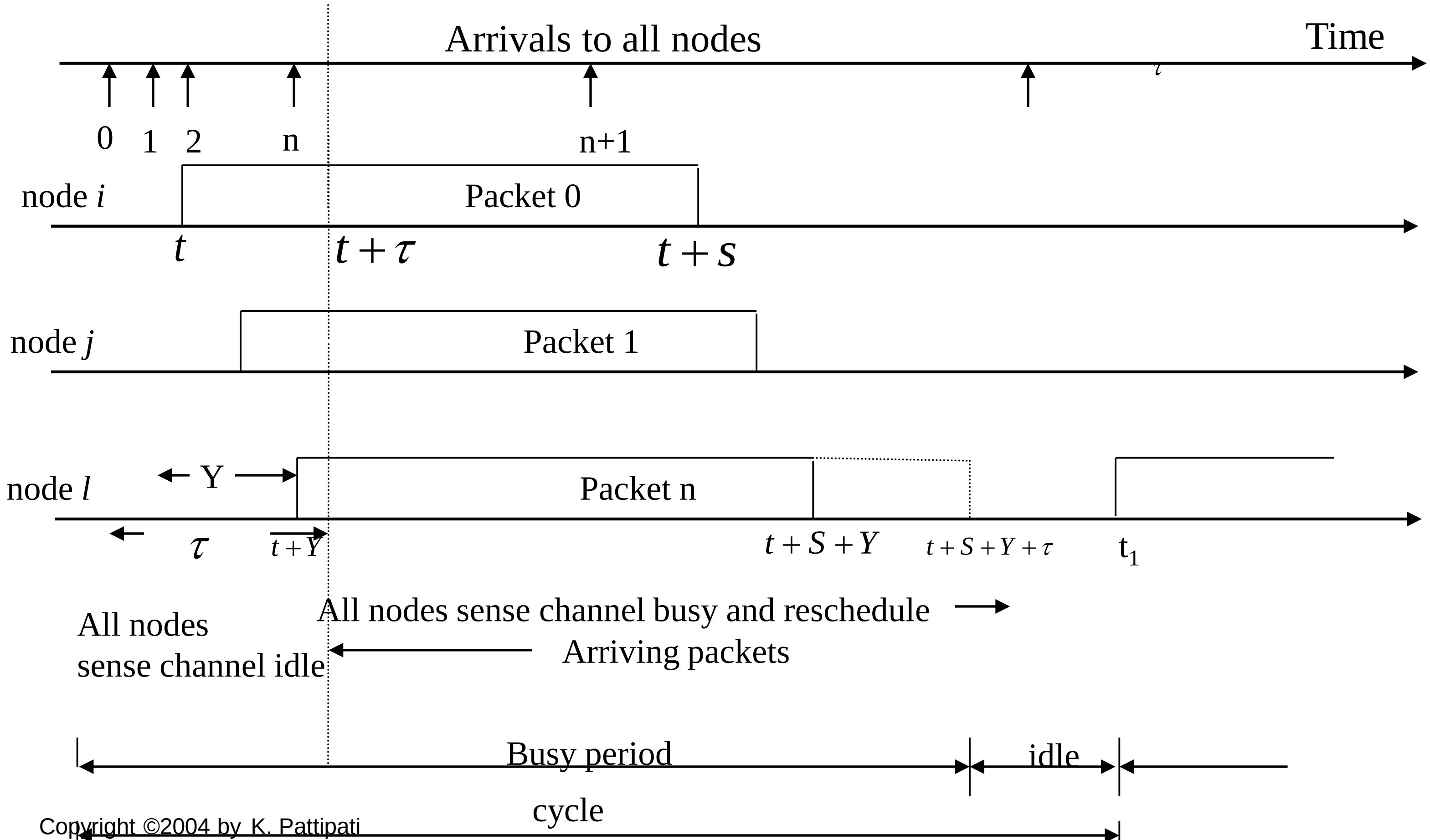
- Model assumptions:

  1. Number of users (nodes) is infinite and the arrival process is Poisson.
  2. Propagation and detection delay is τ seconds.
  3. All packets have the same length and the same transmission time, S.
  4. At any point in time, each node has at most one packet ready for transmission, including any previously collided packets.
  5. Carrier sensing takes place immediately (instantaneous feedback)
  6. Noise-free channel ➔ failure of transmission is due to collisions only. Collision occurs whenever two packets overlap.

Analysis of Unslotted CSMA - 2

- Unsuccessful and successful busy periods for nonpersistent CSMA:

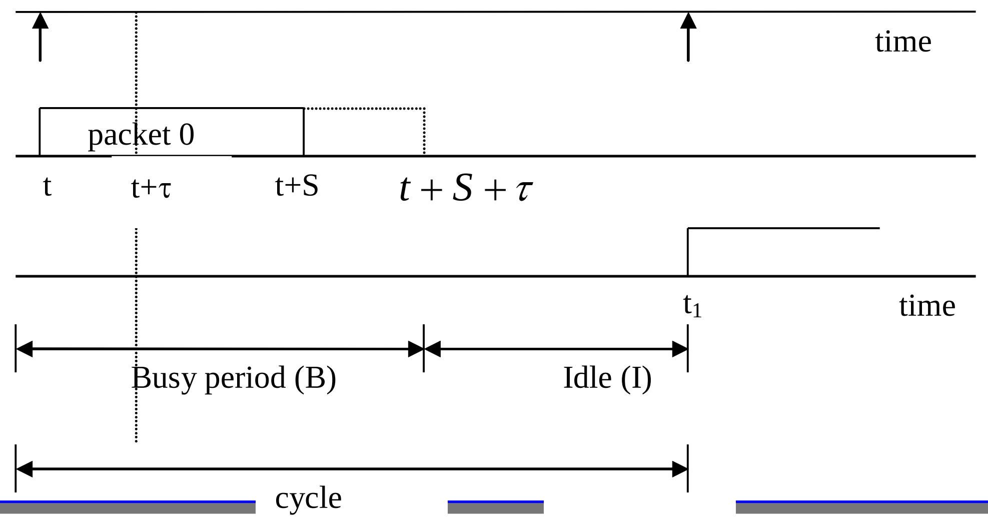
unsuccessful busy period:



Analysis of Unslotted CSMA - 3

- Packet *0* arrives at the reference node at time *t*. Since the channel is sensed idle, the packet is transmitted immediately.
- Packets 1,2,…, n do not know the existence of packet *o*. Let $t+Y$ be the time at which the last packet (in this case *n*) arrives before $t+\tau$.
- After $t+\tau$, nodes know that channel is busy. So, they reschedule packets for a later time (e.g., packet $n+1$ knows that it has to reschedule).
- Packet *n* transmission ends by time $t+Y+S$ and all nodes know about it by $t+Y+S+\tau$.

Successful busy period:



Analysis of Unslotted CSMA - 4

No arrival in $(t, t+\tau) \Rightarrow$ *no collisions occur,* $Y = 0$

So, the vulnerable period for CSMA = propagation and detection delay
(Recall that for pure and slotted Aoha it was 2*S* and *S*, respectively.)

$$\textit{Throughput}\ \rho = \frac{\text{Time over which useful work is performed}}{\text{cycle time}}$$

$$= \frac{\overline{U}}{\overline{B} + \overline{I}}$$

$\overline{U}$ = *average time during a cycle where packets are successfully transmitted.*

1) $\overline{U} = S.\ prob\{packet\ o\ is\ a\ good\ transmission\}$

$$= S.\ prob\{\ o\ arrivals\ in\ (t, t+\tau\ )\}$$

$$= S.e^{-G\tau/S};$$

$\frac{G}{S}$ = *attempt rate (new + retransmissions) per packet transmission time*

Analysis of Unslotted CSMA - 5

2) Busy period length

$$B = Y + S + \tau$$

know that

$$Y = \begin{cases} 0 & \text{if successful transmission with prob } e^{-G\tau/S} \\ \text{some random variable, otherwise with prob } [1-e^{-G\tau/S}] \end{cases}$$

Distribution of Y :

$\mathrm{t + Y = time\ of\ last\ arrival\ of\ a\ packet\ in\ (t, t + \tau)}$

$\Rightarrow$ no packet in $(\mathrm{t + Y, t + \tau})$

$$F_Y(y) = prob\{Y \le y\} = prob\{no\ arrivals\ in\ (t + y; t + \tau)\}$$
$$= e^{-G(\tau - y)/s} \quad for\ 0 \le y \le \tau$$

$$\boxed{\bar{Y} = \int_0^\tau [1 - F_Y(y)]dy = \tau - \frac{S}{G}[1 - e^{-G\tau/S}]}$$

$$so, \quad \bar{B} = 2\tau + S - \frac{S}{G}[1 - e^{-G\tau/S}]$$

$$as\ G \to 0,\ \bar{B} \to \tau + S\ as\ it\ should$$

Analysis of Unslotted CSMA - 6

3) *Mean Idle period,* $\bar{I}$

$$Inter-arrival\ times\ are\ exponential\ with\ mean\ \frac{S}{G}$$

$$so,\quad \bar{I} = \frac{S}{G}$$

$$so,\ \rho = \frac{\bar{U}}{\bar{B}+\bar{I}} = \frac{Se^{-G\tau/S}}{S[1+\frac{2\tau}{S}] + e^{-G\tau/S}\frac{S}{G}}$$

If we let $\beta = \frac{\tau}{S}$ we have

$$\boxed{\rho = \frac{Ge^{-\beta G}}{G(1+2\beta)+e^{-\beta G}}}$$

For small β, $e^{-\beta G} \approx 1-\beta G$

$$\boxed{\rho = \frac{e^{-\beta G}}{[\frac{1}{G}+(1+\beta)]}}$$

Analysis of Unslotted CSMA - 7

- The throughput has a maximum at

$$\frac{d\rho}{dG} = 0 \Rightarrow -\beta e^{-\beta G}[\frac{1}{G} + (1+\beta)] + \frac{1}{G^2} e^{-\beta G} = 0$$

$$or \quad \frac{1}{G^2} - \frac{\beta}{G} - \beta - \beta^2 = 0$$

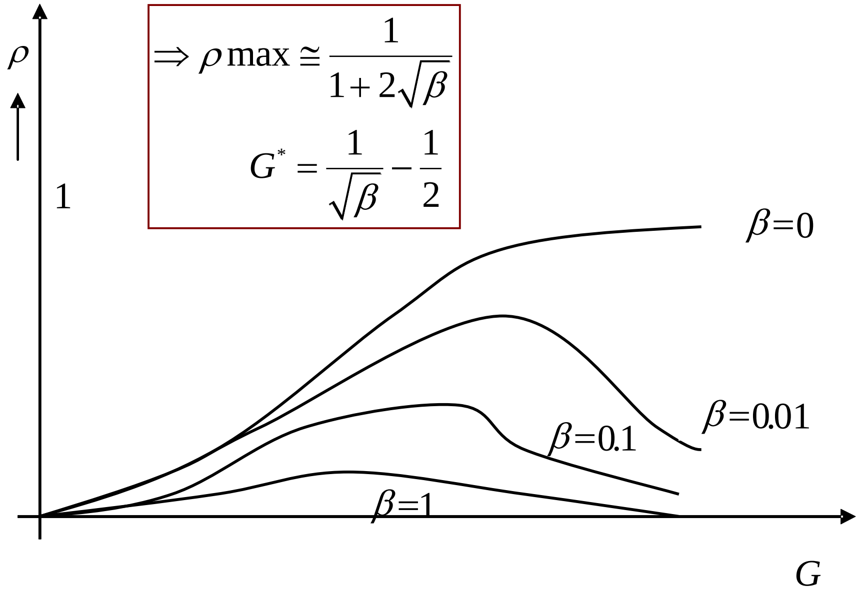
$$(\beta + \beta^2)G^2 + \beta G - 1 = 0$$

$$\beta \text{ small} \Rightarrow \quad G^2 + G - \frac{1}{\beta} \approx 0$$

$$G = \frac{-1 + \sqrt{1 + 4/\beta}}{2} \approx \beta^{-1/2} - 1/2 \boxed{\Rightarrow \rho \max \cong \frac{1}{1 + 2\sqrt{\beta}}}$$

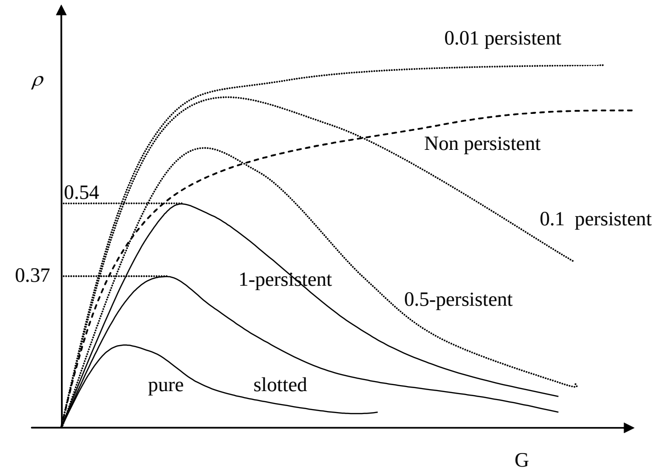
$$\boxed{Note: \quad \beta = 0 \Rightarrow \rho = \frac{G}{1+G} \quad \text{and } \rho_{max} = 1 \text{ at } G = \infty}$$

Performance of Unslotted CSMA

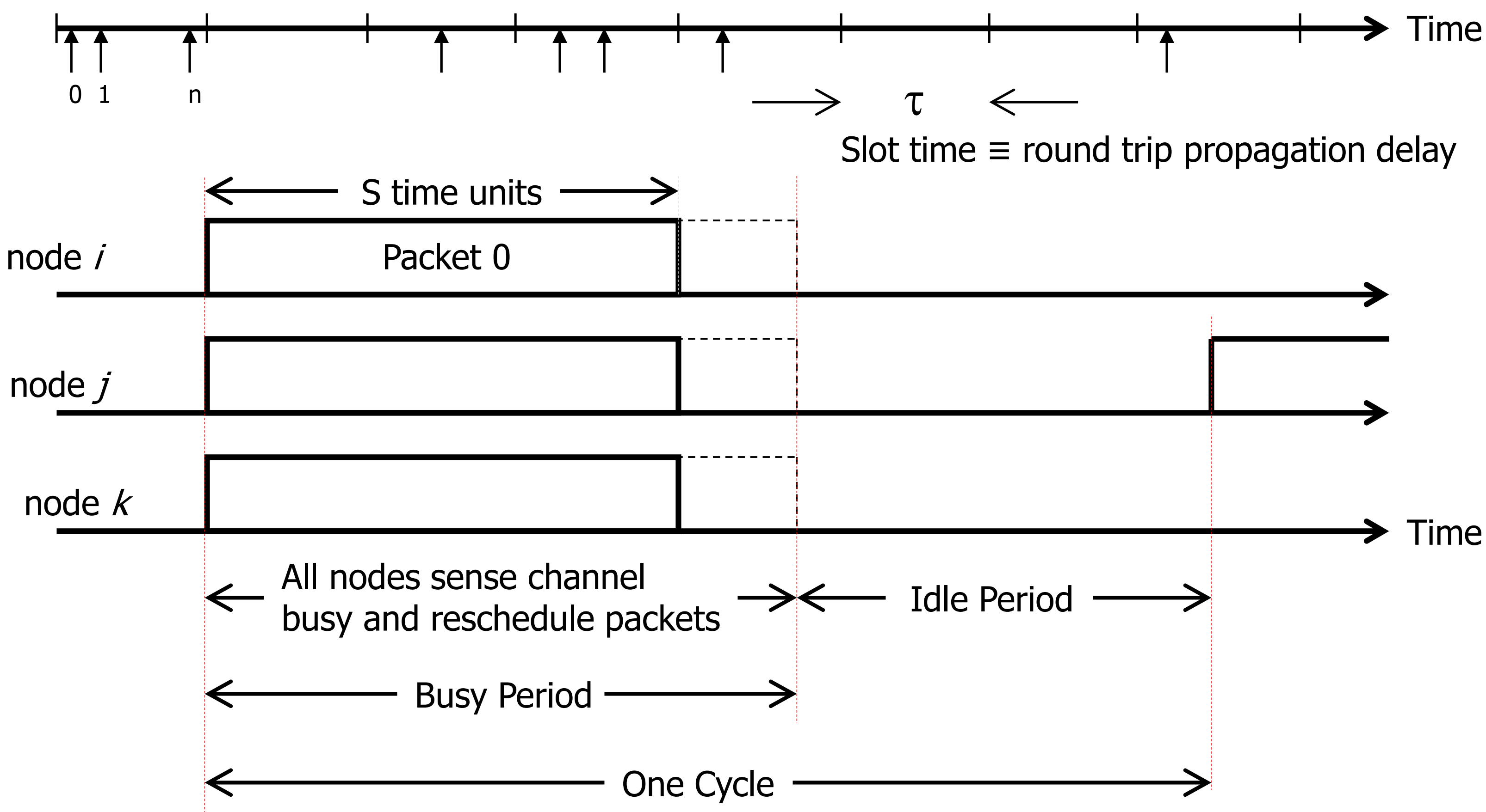


$$\Rightarrow \rho\max \cong \frac{1}{1+2\sqrt{\beta}}$$

$$G^* = \frac{1}{\sqrt{\beta}} - \frac{1}{2}$$



Slotted Non-Persistent CSMA -1



- ❑ S is an integer multiple of slot length τ

Slotted Non-Persistent CSMA - 2

$$\text{Throughput } \rho = \frac{\text{Time over which useful work is done}}{\text{Cycle Time}} = \frac{\bar{U}}{\bar{B}+\bar{I}}$$

$$\bar{U} = S \,.\, P\,\{\text{ packet is a good transmission }\} = S.P_s$$

$$P_s = \text{ prob }\{\text{one arrival in } (0,\tau) \mid \text{an arrival in } (0,\tau)\}$$

$$\text{Attempt rate} = \frac{\text{Ave. \# of attempted transmissions}}{\text{Channel Time}} = \frac{G}{S}$$

$$P_s = \frac{\left(\frac{G}{S}\tau\right)e^{\frac{-G\tau}{S}}}{1-e^{\frac{-G\tau}{S}}} = \frac{\beta G\, e^{-\beta G}}{1-e^{-\beta G}}$$

So, $$\bar{U} = S\left[\frac{\beta G\, e^{-\beta G}}{1-e^{-\beta G}}\right]$$

Average busy period: $\bar{B} = S + \tau = S(1+\beta)$

Average idle period : $\bar{I} = \tau \sum_{j=0}^{\infty} j p_j$; $j =$ length of idle period in slots

Slotted Non-Persistent CSMA - 3

$j = 0 \;\Rightarrow\;$ *an arrival in the last slot of busy period*

$$p_j = (e^{-\beta G})^j (1 - e^{-\beta G}) \quad ; \quad j \geq 0$$

$$\text{so,} \qquad \bar{I} = \tau \sum_{j=1}^{\infty} j (e^{-\beta G})^j (1 - e^{-\beta G}) = \frac{\tau e^{-\beta G}}{1 - e^{-\beta G}}$$

$$\text{so,} \qquad \rho = \frac{s \frac{\beta G e^{-\beta G}}{1 - e^{-\beta G}}}{\frac{\tau e^{-\beta G}}{1 - e^{-\beta G}} + s + \tau} = \frac{\beta G\, e^{-\beta G}}{\beta e^{-\beta G} + (1+\beta)(1 - e^{-\beta G})} = \frac{\beta G\, e^{-\beta G}}{1 + \beta - e^{-\beta G}}$$

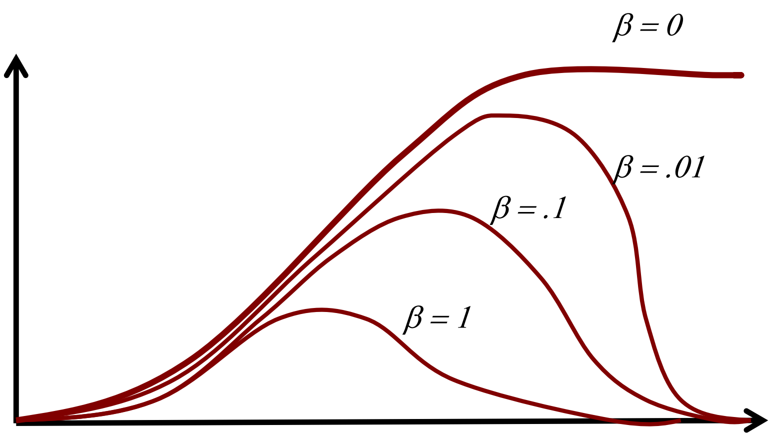
for small β,

$$\rho \approx \frac{G\, e^{-\beta G}}{1+G} = \frac{e^{-\beta G}}{(\frac{1}{G} + 1)} > \rho_{unslotted}$$

For small β, *max. at* $G = \sqrt{\frac{2}{\beta}}$

$$\Rightarrow \rho_{\max} = \frac{1}{1 + \sqrt{2\beta}} \ldots proof\;\; soon$$

Similarly, one can derive results for p-persistent CSMA (See Kleinrock and Tobagi, *IEEE T-Comm, Dec. 1975 & Oct. 1977*)



Slotted Non-Persistent CSMA (4)

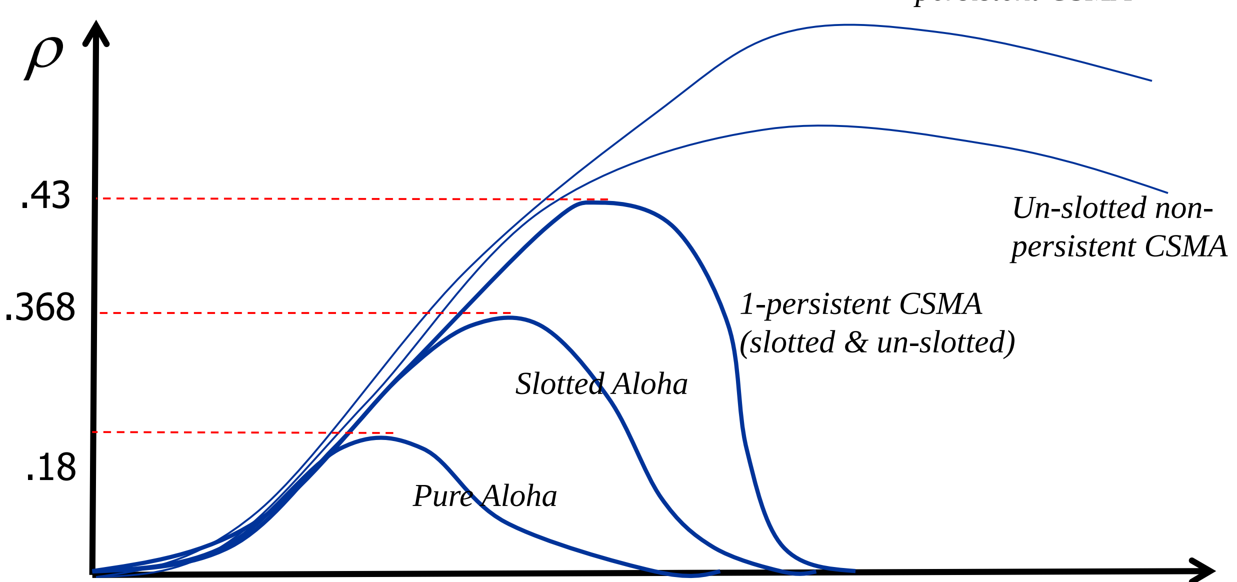
For 1-persistent CSMA: (Unslotted)

$$\rho = \frac{G\left[1+G+\beta G\left(1+G+\frac{\beta G}{2}\right)\right]e^{-G(1+2\beta)}}{G(1+2\beta)-(1-e^{-\beta G})+(1+\beta G)e^{-G(1+\beta)}}$$

For 1-persistent CSMA: (Slotted)

$$\rho = \frac{Ge^{-G(1+\beta)}\left[1+\beta-e^{-\beta G}\right]}{(1+\beta)(1-e^{-\beta G})+\beta e^{-G(1+\beta)}}$$

Typical throughput Curves for β = .01



Stabilization of CSMA - 1
(Pseudo Bayesian Algorithm)

For slotted CSMA

$$\rho = \frac{\beta G\, e^{-\beta G}}{1+\beta - e^{-\beta G}}$$

$$\frac{d\rho}{dG} = 0 \quad \Rightarrow \quad \frac{\left(\beta\, e^{-\beta G} - \beta^2 G\, e^{-\beta G}\right)\left(1+\beta - e^{-\beta G}\right) - \beta^2 G\, e^{-2\beta G}}{\left(1+\beta - e^{-\beta G}\right)^2} = 0$$

$$\Rightarrow \quad \left(\beta\, e^{-\beta G} - \beta^2 G\, e^{-\beta G}\right)\left(1+\beta\right) - \beta\, e^{-2\beta G} = 0$$

$$\Rightarrow \quad \beta(1-\beta G)(1+\beta) = \beta\, e^{-\beta G} \approx \beta\left[1 - \beta G + \frac{\beta^2 G^2}{2}\right]$$

$$\Rightarrow \quad \left(1-\beta G\right)\beta^2 - \frac{\beta^3 G^2}{2} = 0 \quad \text{or} \quad 1 - \beta G - \frac{\beta G^2}{2} = 0$$

$$\Rightarrow \quad G^2 + 2G - \frac{2}{\beta} = 0 \quad \text{or} \quad G \cong \sqrt{\frac{2}{\beta}}$$

or, at $\beta G = \sqrt{2\beta}$; $\rho_{\max} \cong \dfrac{1}{1+\sqrt{2\beta}}$

Stabilization of CSMA - 2
(Pseudo Bayesian Algorithm)

To achieve maximum,

estimate $\hat{n}_k$

set prob of retransmission $\gamma_k = \dfrac{\sqrt{2\beta}}{\hat{n}_k}$

in fact, set $\gamma_k = \min\left(\dfrac{\sqrt{2\beta}}{\hat{n}_k}, \sqrt{2\beta}\right)$

update $\hat{n}_k$ via

$$\hat{n}_{k+1} = \begin{cases} \hat{n}_k(1-\gamma_k) + \rho\beta & \text{for idle} \\ \hat{n}_k(1-\gamma_k) + \rho(1+\beta) & \text{for success} \\ \hat{n}_k + 2 + \rho(1+\beta) & \text{for collision} \end{cases}$$

Splitting algorithm don't do any better in this case

CSMA/CD

- CSMA/CD:
  - "Listen while transmit"
  - "Enables the detection of a collision shortly after it arrives and thus abort flawed packets promptly"
  - "Minimizes channel time occupied by unsuccessful transmissions"
  - Transmits a jamming signal when collision occurs and backs-off

Slotted or Unslotted

Slotted CSMA/CD - 1

$G =$ *offered traffic per packet time, S*

$g = \dfrac{G\tau}{S} =$ *attempt rate per propagation delay* $= G\beta$

Consider state transitions at the end of idle slots

<u>If</u> no transmission occurs, the next idle slot ends after time τ

<u>If</u> one transmission occurs, the next idle slot ends after time $\tau + S$

<u>If</u> a collision occurs, the next idle slot ends after 2τ*, i.e., nodes must hear an idle slot after the collision to know that it is safe to transmit*

So, expected length of interval between state transitions

$$E(Interval) = \tau e^{-g} + ge^{-g}(\tau + S) + \left[1-(1+g)e^{-g}\right]2\tau$$

$$= \tau + ge^{-g}S + \tau\left[1-(1+g)e^{-g}\right]$$

$$= S\left[\beta + ge^{-g} + \beta\left[1-(1+g)e^{-g}\right]\right]$$

Expected numbers of arrivals between state transitions

$$\bar{n} = \rho\left[\beta + ge^{-g} + \beta\left[1-(1+g)e^{-g}\right]\right]$$

Expected Drift in $n = \bar{n} - P_s = \bar{n} - ge^{-g}$

Slotted CSMA/CD - 2

So,

$$\rho < \frac{ge^{-g}}{\beta + ge^{-g} + \beta\left[1-(1+g)e^{-g}\right]}$$

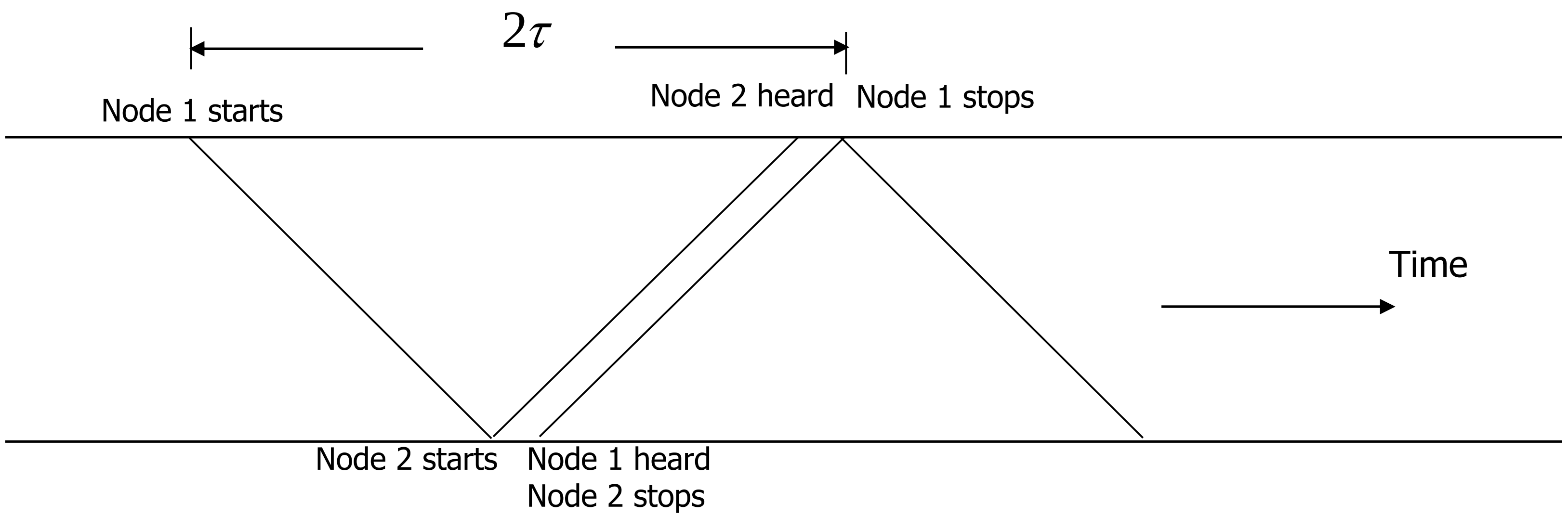
$$\rho_{\max} \ \ at\ g = 0.77$$

$$\rho_{\max} \approx \frac{1}{1+3.31\beta} \quad compare\ with\ \rho_{\max} = \frac{1}{1+\sqrt{2\beta}}\ for\ CSMA$$

For β *small, can get* $\rho_{\max} \approx 1$

Un-slotted CSMA/CD - 1

(used extensively in Local Area Networks)



- Two complications:

1. Random times of collision detections
2. 'When you hear' depends on 'where you are located' on the bus

Un-slotted CSMA/CD - 2
(used extensively in Local Area Networks)

- But, can get a lower bound on throughput as follows:
- Assume that each node initiates transmission whenever the channel is idle
  $\frac{G}{S} \equiv$ the overall Poisson intensity
- All nodes sense the beginning of an idle period at most τ seconds after the end of a transmission
- Expected time to the beginning of the next transmission is at most an additional S/G This next packet will collide with some other packets with probability $1-\exp(-\tau G/S)$
- Colliding packets will cease transmission after at most 2τ seconds
- Packet will be successfully transmitted with probability $\exp(-\tau G/S)$ and will occupy S seconds

So, $$\rho > \frac{\exp(-\tau G/S)S}{\tau + S/G + 2\tau(1-\exp(-\beta G)) + \exp(-\beta G)S}$$

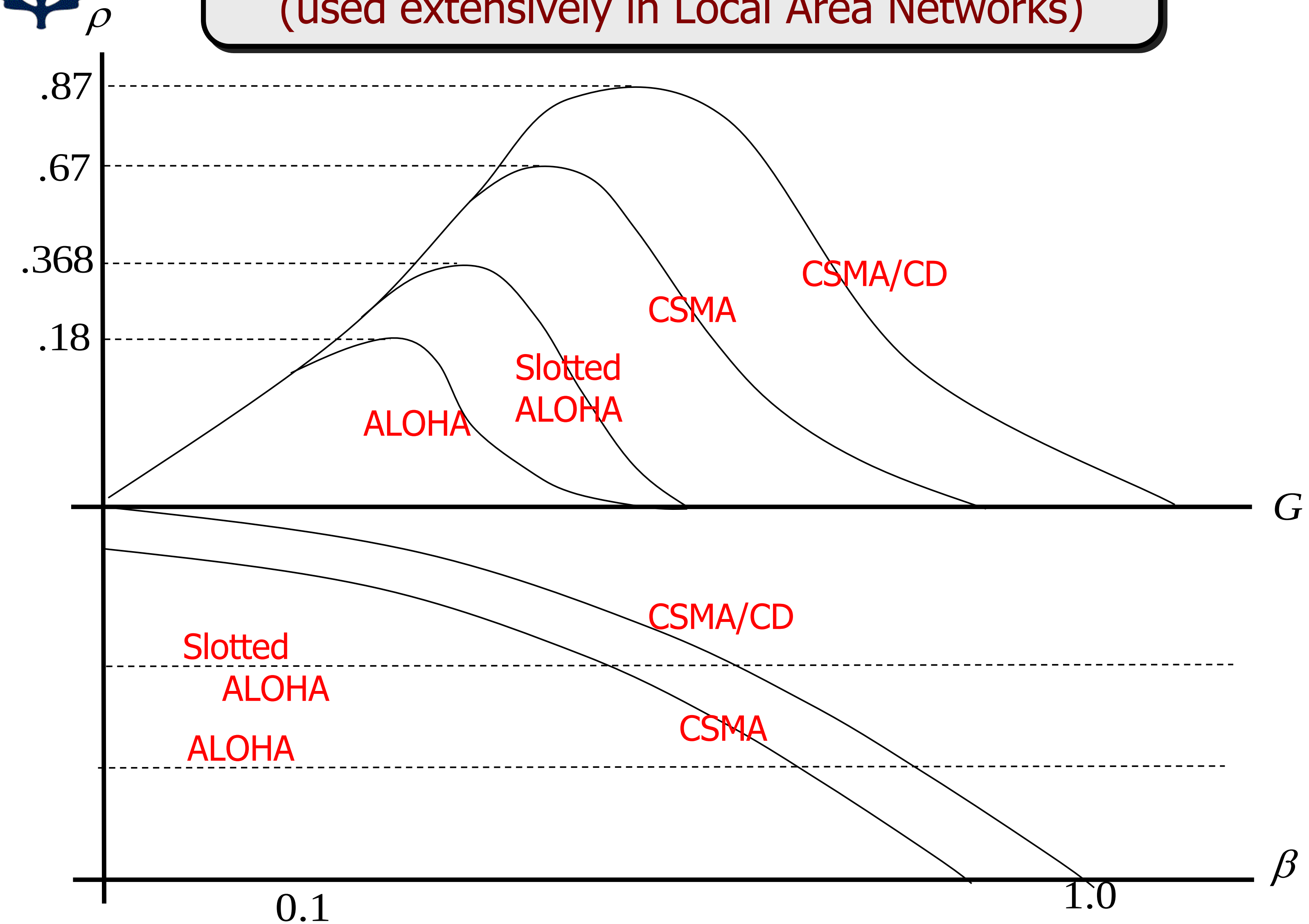
$$\rho > \frac{\exp(-\beta G)}{\beta + S/G + 2\beta(1-\exp(-\beta G))}$$

$$Max\ at \quad \beta G = \left(\sqrt{13}-1\right)/6 \approx 0.43$$

$$So, \qquad \rho > \frac{1}{1+6.2\beta} \qquad Compare\ \rho_{\max} = \frac{1}{1+\sqrt{2\beta}} \quad for\ CSMA$$

Un-slotted CSMA/CD - 3

(used extensively in Local Area Networks)



Multi-access Reservations - 1

- Basic idea:
  - Use short packets to reserve slots for longer data packets
  - Suppose data packets require 1 time unit and reservation packets require v time units /slots
  - If ρ_r is the maximum throughput of reservation packets per reservation slot, then,

$$average\ time\ per\ reservation = v/\rho_r$$

$$total\ time = 1 + v/\rho_r$$

$$maximum\ throughput\ \rho_{\max} = \frac{1}{1 + v/\rho_r}$$

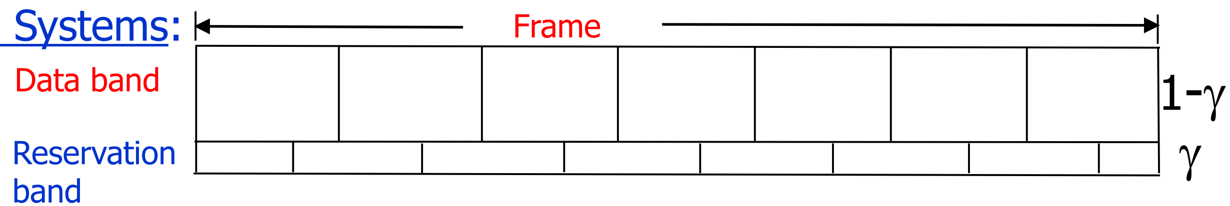
$$v = 0.01,\ and\ Slotted\ ALOHA \Rightarrow \rho_{\max} = \frac{1}{1+ve} = 0.9735$$

- Suppose, we use reservation packets also to send some data, then,

$$total\ time\ = 1 - v + v/\rho_r$$

$$so,\ \rho_{\max} = \frac{1}{1 + v\left(1/\rho_r - 1\right)} \overset{Slotted\ ALOHA}{=} \frac{1}{1 + v\left(e - 1\right)} \rightarrow 1\ as\ v \rightarrow 0$$

- Satellite Reservation Systems:



Multi-access Reservations - 2

- Round trip propagation delay = 2β
- Fraction γ of the available bandwidth is used for reservations
- TDM is used within this bandwidth $\Rightarrow$ one reservation packet per round trip delay period 2β

Let $$v = \frac{\text{reservatio n packet length}}{\text{data packet length}}$$

$$\gamma = \frac{mv}{2\beta}$$

- An arriving packet waits β time units until the beginning of reservation interval
- $\textit{Transmission of reservation packet } = \dfrac{2\beta}{m} \textit{ time units}$
- $\textit{Round trip delay } = 2\beta \textit{ (confirmation of reservation)}$

$$\Rightarrow \textit{ A delay of } \beta + \frac{2\beta}{m} + 2\beta = 3\beta + \frac{2\beta}{m} \textit{ time units}$$

Multi-access Reservations - 3

- *The transmission is like M/G/1 queue* $X \rightarrow \frac{X}{1-\gamma}, \rho = \frac{\lambda \bar{X}}{1-\gamma}$

$$W = 3\beta + \frac{2\beta}{m} + \frac{\lambda \overline{X^2}}{2(1-\gamma)^2(1-\rho)}$$

$$= 3\beta + \frac{2\beta}{m} + \frac{\lambda \overline{X^2}}{2(1-\gamma)(1-\gamma-\lambda\bar{X})}$$

$\exists$ *many variations (e.g., the first packet makes reservations for subsequent packets)*

Summary

- Carrier Sense Multipler Access (CSMA)
- Pseudo Bayesian Algorithm for CSMA
- CSMA/CD (Collision Detection)
- Multi-access Reservations