Solution 2

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ECE 6141
Neural Networks for Classification and Optimization

- Problem 1

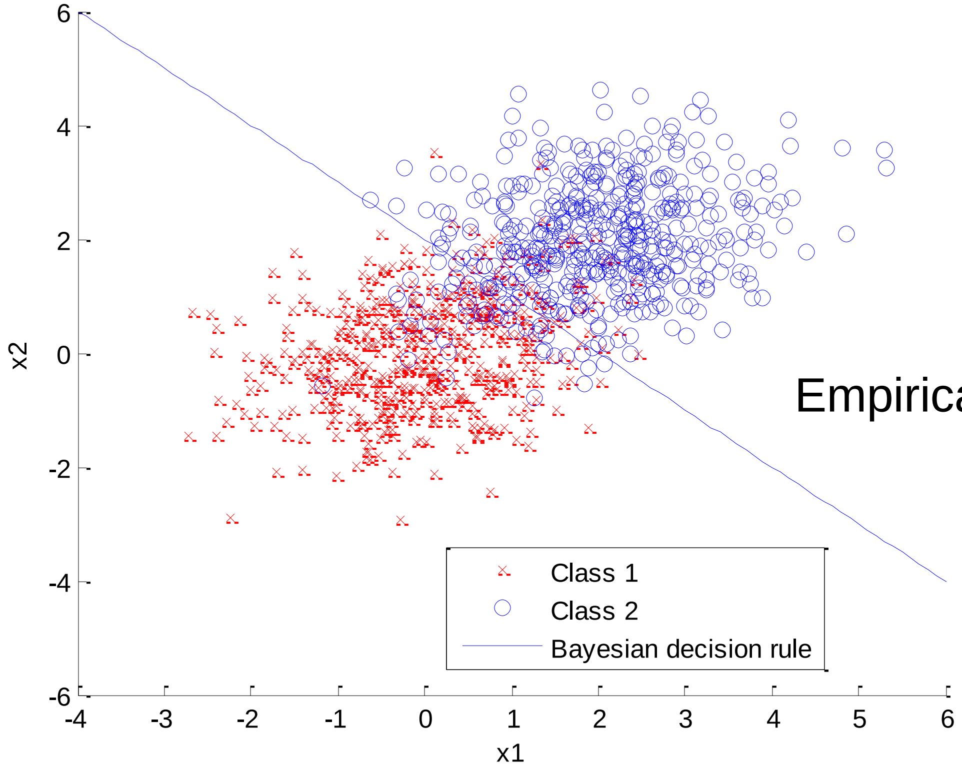
$$\text{Recall } g(x)=\sigma^2[g_1(x)-g_2(x)]=(\mu_1-\mu_2)x-[\frac{1}{2}(\mu_1^2-\mu_2^2)-\sigma^2\ln\frac{P(\omega_1)}{P(\omega_2)}]\underset{C_2}{\overset{C_1}{>}}0$$

$$\text{Assuming equal priors, } g(x)=x\underset{C_2}{\overset{C_1}{>}}0$$

$$P_e=\Phi(-\frac{d'}{2})=\Phi(-10)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-10}e^{-\frac{u^2}{2}}du\approx\frac{1}{\sqrt{2\pi}10}e^{-50}\approx 7.6946*10^{-24}$$

$$\text{using for large } x, \Phi(-x)\approx\frac{1}{\sqrt{2\pi}\,|x|}e^{-\frac{x^2}{2}}$$

- Problem 2



Empirical error is ~10%

- Problem 2

$$\underline{\mu}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \underline{\mu}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\Sigma_1 = \Sigma_2 = \Sigma = \begin{bmatrix} 1 & 0.25 \\ 0.25 & 1 \end{bmatrix} \Rightarrow \Sigma^{-1} = \begin{bmatrix} 16/15 & -4/15 \\ -4/15 & 16/15 \end{bmatrix}$$

Discriminant $g(\underline{x}) = \left(\underline{\mu}_1 - \underline{\mu}_2\right)^T \Sigma^{-1}\underline{x} - \frac{1}{2}\left(\underline{\mu}_1^T \Sigma^{-1}\underline{\mu}_1 - \underline{\mu}_2^T \Sigma^{-1}\underline{\mu}_2\right)$ **assuming equal priors**

$$= -1.6x_1 - 1.6x_2 + 3.2$$

$\Rightarrow class\ 1\ if\ \ g'(\underline{x}) = x_1 + x_2 - 2 \le 0; else\ class\ 2.$

$p(g(\underline{x}) \mid z=1) = N(3.2, 6.4);\ p(g(\underline{x}) \mid z=2) = N(-3.2, 6.4)$

$$P_e = \frac{1}{2}\left[\underbrace{\int_0^{\infty} N(g; 3.2, 6.4)dg}_{decision=2} + \underbrace{\int_{-\infty}^{0} N(g; -3.2, 6.4)dg}_{decision=1}\right]$$

$$= \int_{-\infty}^{0} N(g; 3.2, 6.4)dg = \Phi(-1.2649) = 0.10295$$

$Alternately: p(g'(\underline{x}) \mid z=1) = N(-2, 2.5);\ p(g'(\underline{x}) \mid z=2) = N(2, 2.5)$

$$P_e = \frac{1}{2}\left[\int_0^{\infty} N(g; -2, 2.5)dg + \int_{-\infty}^{0} N(g; 2, 2.5)dg\right] = 0.10295$$

- Problem 2 (assume equal priors)

$$\Lambda = \begin{bmatrix} 0 & 1 \\ 0.005 & 0 \end{bmatrix}$$

$$\Rightarrow class\ 1\ if\ \ P(z=2\,|\,\underline{x}) \le 0.005P(z=1\,|\,\underline{x}); else\ class\ 2$$

$$\Rightarrow class\ 1\ if\ \ P(z=1\,|\,\underline{x}) \ge 200P(z=2\,|\,\underline{x}); else\ class\ 2.$$

$$\Rightarrow class\ 1\ if\ \ P(\underline{x}\,|\,z=1) \ge 200P(\underline{x}\,|\,z=2); else\ class\ 2.$$

$$\Rightarrow class\ 1\ if\ \ \underline{\mu}_2^T \Sigma^{-1} \underline{x} \le \frac{\underline{\mu}_2^T \Sigma^{-1} \underline{\mu}_2}{2} - \ln 200; else\ class\ 2$$

$$\Rightarrow class\ 1\ if\ \ g(\underline{x}) = 1.6x_1 + 1.6x_2 + 2.0983 \le 0; else\ class\ 2$$

$$p(g(\underline{x})\,|\,z=1) = N(g; 2.0983, 6.4)$$

$$p(g(\underline{x})\,|\,z=2) = N(g; 8.4983, 6.4)$$

$$ECM = \frac{1}{2}[0.005 * \underbrace{P\{(g(\underline{x})\,|\,z=1) \ge 0\}}_{decision=2} + \underbrace{P\{(g(\underline{x})\,|\,z=2) \le 0\}}_{decision=1}]$$

$$= \frac{1}{2}[0.7966 * 0.005 + 0.00039077) = 0.0022$$

$$Empirical\ Risk = \frac{1}{N}\sum_{k=1}^{N} \min[P(z=2\,|\,\underline{x}_k), 0.005P(z=1\,|\,\underline{x}_k)]$$

- Problem 2 $\Lambda = \begin{bmatrix} 0 & 1000 \\ 1 & 0 \end{bmatrix}$

$$\Rightarrow class\,1\, if\;\; 1000P(z=2\,|\,\underline{x}) \le P(z=1\,|\,\underline{x}); else\; class\; 2$$

$$\Rightarrow class\,1\, if\;\; P(z=1\,|\,\underline{x}) \ge 1000P(z=2\,|\,\underline{x}); else\; class\; 2.$$

$$\Rightarrow class\,1\, if\;\; P(\underline{x}\,|\,z=1) \ge 1000P(\underline{x}\,|\,z=2); else\; class\; 2.$$

$$\Rightarrow class\,1\, if\;\; \ln P(\underline{x}\,|\,z=1) \ge \ln 1000 + \ln P(\underline{x}\,|\,z=2); else\; class\; 2.$$

$$\Rightarrow class\,1\, if\; -\frac{1}{2}(\underline{x}-\underline{\mu}_1)^T \Sigma^{-1}(\underline{x}-\underline{\mu}_1) \ge 6.9078 - \frac{1}{2}(\underline{x}-\underline{\mu}_2)^T \Sigma^{-1}(\underline{x}-\underline{\mu}_2); else\; class\; 2.$$

$$\Rightarrow class\,1\, if\; -\underline{\mu}_2^T \Sigma^{-1}\underline{x} \ge 6.9078 - \frac{\underline{\mu}_2^T \Sigma^{-1}\underline{\mu}_2}{2} = 6.9078 - 3.2 = 3.7078; else\; class\; 2.$$

$$\Rightarrow class\,1\, if\; -1.6x_1 - 1.6x_2 \ge 3.7078; else\; class\; 2.$$

$$\Rightarrow class\,1\, if\;\; g(\underline{x}) = x_1 + x_2 + 2.3174 \le 0; else\; class\; 2.$$

$$p(g(\underline{x})\,|\,z=1) = N(g; 2.3174, 2.5);\; p(g(\underline{x})\,|\,z=2) = N(g; 6.3174, 2.5)$$

Class 1	Class 2
37	**0**
463	500

$$ECM = \frac{1}{2}[\underbrace{P\{(g(\underline{x})\,|\,z=1) > 0\}}_{decision=2} + 1000\underbrace{P\{(g(\underline{x})\,|\,z=2) \le 0\}}_{decision=1}]$$

$$= \frac{1}{2}[0.9286 + 1000 * 0.00003) = 0.46445$$

$$Empirical\;\; Risk = \frac{1}{N}\sum_{k=1}^{N}\left[1000 * P(decision=1, class\,2) + P(decision=2, class\,1)\right] = 0.463$$

- Problem 2 $\Lambda = \begin{bmatrix} 0 & 1 \\ 10 & 0 \end{bmatrix}$

$$\Rightarrow class\,1\,if\;\; P(z=2\,|\,\underline{x}) \le 10P(z=1\,|\,\underline{x});\, else\; class\; 2$$

$$\Rightarrow class\,1\,if\;\; P(z=1\,|\,\underline{x}) \ge 0.1P(z=2\,|\,\underline{x});\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\;\; P(\underline{x}\,|\,z=1) \ge 0.1P(\underline{x}\,|\,z=2);\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\;\; \ln P(\underline{x}\,|\,z=1) \ge -2.3026 + \ln P(\underline{x}\,|\,z=2);\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\; -\frac{1}{2}(\underline{x}-\underline{\mu}_1)^T \Sigma^{-1}(\underline{x}-\underline{\mu}_1) \ge -2.3026 - \frac{1}{2}(\underline{x}-\underline{\mu}_2)^T \Sigma^{-1}(\underline{x}-\underline{\mu}_2);\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\; -\underline{\mu}_2^T \Sigma^{-1}\underline{x} \ge -2.3026 - \frac{\underline{\mu}_2^T \Sigma^{-1}\underline{\mu}_2}{2} = -2.3026 - 3.2 = -5.5026;\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\; -1.6x_1 - 1.6x_2 \ge -5.5026;\, else\; class\; 2.$$

$$\Rightarrow class\,1\,if\;\; g(\underline{x}) = x_1 + x_2 - 3.4391 \le 0;\, else\; class\; 2.$$

$$p(g(\underline{x})\,|\,z=1) = N(g;-3.4391, 2.5);\; p(g(\underline{x})\,|\,z=2) = N(g;0.5609, 2.5)$$

$$ECM = \frac{1}{2}[10 * \underbrace{P\{(g(\underline{x})\,|\,z=1) > 0\}}_{decision=2} + \underbrace{P\{(g(\underline{x})\,|\,z=2) \le 0\}}_{decision=1}]$$

$$= \frac{1}{2}[10 * 0.0148 + 0.36139) = 0.2547$$

Class 1	Class 2
492	**160**
8	340

$$Empirical\;\; Risk = \frac{1}{N}\sum_{k=1}^{N}\left[P(decision=1, class\,2) + 10 * P(decision=2, class\,1)\right] = 0.24$$

- Problem 3

$Q =$ Quantity to buy

D = Demand with *pdf* $f(D)$ and *distribution* $F(D)$

C = Cost per item

P = Selling price

$$\text{Profit} = \begin{cases} (P-C)Q \text{ if } D > Q \\ (P-C)D \text{ if } D \le Q \end{cases}$$

$$\text{Waste} = \begin{cases} 0 \text{ if } D > Q \\ (Q-D)C \text{ if } D \le Q \end{cases}$$

$$Net\ expected\ \ profit = E_D[\pi(Q)] = \int_Q^\infty (P-C)Qf(D)dD + \int_0^Q (P-C)Df(D)dD - \int_0^Q (Q-D)Cf(D)dD$$

$$Optimal\ Q^* \Rightarrow \frac{dE_D[\pi(Q)]}{dQ} = 0 \Rightarrow \int_{Q^*}^\infty (P-C)f(D)dD - (P-C)Q^* f(Q^*) + (P-C)Q^* f(Q^*) - \int_0^{Q^*} Cf(D)dD = 0$$

$$\Rightarrow (P-C)[1-F(Q^*)] - CF(Q^*) = 0 \Rightarrow (P-C) - PF(Q^*) = 0 \Rightarrow F(Q^*) = \frac{P-C}{P} \Rightarrow Q^* = F^{-1}(\frac{P-C}{P})$$

$$\text{Second derivative: } \frac{d^2E_D[\pi(Q)]}{dQ^2} = -(P-C)f(Q^*) - Cf(Q^*) = -Pf(Q^*) < 0 \Rightarrow \text{ maximum}$$

- Problem 4

$$L(\Delta, p(.)) = -\ln p(\Delta)$$

Two possibilities :

$$p^{BMA}(\Delta) = \sum_m p(\Delta \mid m, D)P(m \mid D)$$

$$p^m(\Delta) = p(\Delta \mid m, D)$$

$$E_{p^{BMA}}[L(\Delta, p^{BMA})] = -\int_\Delta p^{BMA}(\Delta) \ln p^{BMA}(\Delta) d\Delta$$

$$E_{p^{BMA}}[L(\Delta, p^m)] = -\int_\Delta p^{BMA}(\Delta) \ln p^m(\Delta)\, d\Delta$$

$$E_{p^{BMA}}[L(\Delta, p^{BMA}) - L(\Delta, p^m)] = -\int_\Delta p^{BMA}(\Delta) \ln \frac{p^{BMA}(\Delta)}{p^m(\Delta)} d\Delta = -KL(p^{BMA}(\Delta) \,\|\, p^m(\Delta)) \le 0$$

$$Why?-\int_\Delta p^{BMA}(\Delta) \ln \frac{p^{BMA}(\Delta)}{p^m(\Delta)} d\Delta = \int_\Delta p^{BMA}(\Delta) \ln \frac{p^m(\Delta)}{p^{BMA}(\Delta)} d\Delta \le \ln \int_\Delta p^{BMA}(\Delta) \frac{p^m(\Delta)}{p^{BMA}(\Delta)} d\Delta = \ln 1 = 0$$

Re*call* $\ln x$ *is concave*

- Problem 5

$$a.\ p(x,y|\underline{\theta}) = p(x|\theta_1)\,p(y|x,\theta_2)$$

$$p(x|\theta_1) = \theta_1^{x}(1-\theta_1)^{1-x}$$

$$p(y|x,\theta_2) = \theta_2^{[(1-x)(1-y)+xy]}(1-\theta_2)^{[(1-x)y+x(1-y)]}$$

$$\therefore\ p(x,y|\underline{\theta}) = \theta_1^{x}(1-\theta_1)^{1-x}\,\theta_2^{[(1-x)(1-y)+xy]}(1-\theta_2)^{[(1-x)y+x(1-y)]}$$

In terms of a 2x2 table, $p(x,y|\underline{\theta})$ is:

$$\begin{array}{cc} & \begin{array}{cc} y=0 & y=1 \end{array} \\ \begin{array}{c} x=0 \\ x=1 \end{array} & \begin{bmatrix} (1-\theta_1)\theta_2 & (1-\theta_1)(1-\theta_2) \\ \theta_1(1-\theta_2) & \theta_1\theta_2 \end{bmatrix} \end{array}$$

$$b.\ p(D|\underline{\theta},M_2) = \prod_{i=1}^{N}\left[\theta_1^{x_i}(1-\theta_1)^{1-x_i}\,\theta_2^{[(1-x_i)(1-y_i)+x_iy_i]}(1-\theta_2)^{[(1-x_i)y_i+x_i(1-y_i)]}\right];\ N=7$$

$$Given:\ \underline{x} = (1,1,0,1,1,0,0);\ \underline{y} = (1,0,0,0,1,0,1)$$

- Problem 5

$$Given: \underline{x} = (1,1,0,1,1,0,0); \underline{y} = (1,0,0,0,1,0,1)$$

$$Let\ n_1 = \sum_{i=1}^{7} x_i = 4; n_0 = 7 - n_1 = 3$$

$$Let\ D_c = \begin{bmatrix} n_{00} & n_{01} \\ n_{10} & n_{11} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

$$J_2 = -\ln p(D|\underline{\theta}, M_2) = -\left[n_1 \ln\theta_1 + n_0 \ln(1-\theta_1) + (n_{00}+n_{11})\ln\theta_2 + (n_{01}+n_{10})\ln(1-\theta_2)\right]$$
$$= -\left[4\ln\theta_1 + 3\ln(1-\theta_1) + 4\ln\theta_2 + 3\ln(1-\theta_2)\right]$$

$$\frac{\partial J_2}{\partial \theta_1} = 0 \Rightarrow \hat{\theta}_{1,MLE} = \frac{4}{7}$$

$$\frac{\partial J_2}{\partial \theta_2} = 0 \Rightarrow \hat{\theta}_{2,MLE} = \frac{4}{7}$$

$$J_2^* = -8\ln\frac{4}{7} - 6\ln\frac{3}{7} = 9.5607 \Rightarrow p(D|\hat{\underline{\theta}}_{MLE}, M_2) = e^{-J_2} = 7.0443\text{x}10^{-5}$$

$$c) Similarly, \underline{\theta} = (\theta_{00}, \theta_{01}, \theta_{10}, \theta_{11})$$

$$J_4 = -\ln p(D|\underline{\theta}, M_4) = -\left[2\ln\theta_{00} + \ln\theta_{01} + 2\ln\theta_{10} + 2\ln(1-\theta_{00}-\theta_{01}-\theta_{10})\right]$$

- Problem 5

$$Given: \underline{x} = (1,1,0,1,1,0,0); \underline{y} = (1,0,0,0,1,0,1)$$

$$Let\ n_1 = \sum_{i=1}^{7} x_i = 4; n_0 = 7 - n_1 = 3$$

$$Let\ D_c = \begin{bmatrix} n_{00} & n_{01} \\ n_{10} & n_{11} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

$$J_4 = -\ln p(D \mid \underline{\theta}, M_4) = -\left[n_{00} \ln \theta_{00} + n_{01} \ln \theta_{01} + n_{10} \ln \theta_{10} + n_{11} \ln \underbrace{(1 - \theta_{00} - \theta_{01} - \theta_{10})}_{\theta_{11}} \right]$$

$$= -\left[2 \ln \theta_{00} + \ln \theta_{01} + 2 \ln \theta_{10} + 2 \ln \underbrace{(1 - \theta_{00} - \theta_{01} - \theta_{10})}_{\theta_{11}} \right]$$

$$Easy\ to\ see\ that\ \hat{\theta}_{00} = \hat{\theta}_{10} = \hat{\theta}_{11} = \frac{2}{7}; \hat{\theta}_{01} = \frac{\hat{\theta}_{00}}{2} = \frac{1}{7}$$

$$J_4^* = 9.4625 \Rightarrow p(D \mid \underline{\hat{\theta}}_{MLE}, M_4) = e^{-J_4} = 7.7712\text{x}10^{-5}$$

- Problem 5

$d) Given: \underline{x} = (1,1,0,1,1,0,0); \underline{y} = (1,0,0,0,1,0,1)$

$$Let\ n_1 = \sum_{i=1}^{7} x_i = 4; n_0 = 7 - n_1 = 3$$

$$Let\ D_c = \begin{bmatrix} n_{00} & n_{01} \\ n_{10} & n_{11} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}; LOOCV: D_{-1} = D_{-5} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}; D_{-2} = D_{-4} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};$$

$$D_{-3} = D_{-6} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}; D_{-7} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$$

$For\ \text{model}\ 2:$

$$(i) For\ D_{-1}\ \&\ D_{-5}: J = -\left[3\ln\theta_1 + 3\ln(1-\theta_1) + 3\ln\theta_2 + 3\ln(1-\theta_2)\right] \Rightarrow \hat{\theta}_1 = \hat{\theta}_2 = \frac{1}{2}$$

$$(ii) For\ D_{-2}\ \&\ D_{-4}: J = -\left[3\ln\theta_1 + 3\ln(1-\theta_1) + 4\ln\theta_2 + 2\ln(1-\theta_2)\right] \Rightarrow \hat{\theta}_1 = \frac{1}{2}; \hat{\theta}_2 = \frac{2}{3}$$

$$(iii) For\ D_{-3}\ \&\ D_{-6}: J = -\left[4\ln\theta_1 + 2\ln(1-\theta_1) + 3\ln\theta_2 + 3\ln(1-\theta_2)\right] \Rightarrow \hat{\theta}_1 = \frac{2}{3}; \hat{\theta}_2 = \frac{1}{2}$$

$$(iv) For\ D_{-7}: -\left[4\ln\theta_1 + 2\ln(1-\theta_1) + 4\ln\theta_2 + 2\ln(1-\theta_2)\right] \Rightarrow \hat{\theta}_1 = \frac{2}{3}; \hat{\theta}_2 = \frac{2}{3}$$

$$L(2) = \ln p(D\,|\,\underline{\theta}, M_2) = 4\ln\frac{1}{2} + 2[\ln\frac{1}{2} + \ln\frac{1}{3}] + 2[\ln\frac{1}{3} + \ln\frac{1}{2}] + 2\ln\frac{1}{3} = 8\ln\frac{1}{2} + 6\ln\frac{1}{3} = -12.1369$$

- Problem 5

$$d)\, Given: \underline{x} = (1,1,0,1,1,0,0);\ \underline{y} = (1,0,0,0,1,0,1)$$

$$Let\ n_1 = \sum_{i=1}^{7} x_i = 4; n_0 = 7 - n_1 = 3$$

$$Let\ D_c = \begin{bmatrix} n_{00} & n_{01} \\ n_{10} & n_{11} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}; LOOCV: D_{-1} = D_{-5} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}; D_{-2} = D_{-4} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix};$$

$$D_{-3} = D_{-6} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}; D_{-7} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$$

For model $4: For\ D_{-7}, \hat{\theta}_{01} = 0 \Rightarrow L(4) = -\infty$ because $p((0,1)\,|\,D_{-7}, M_4) = \ln 0 = -\infty$

So, LOOCV will pick model M_2.

$$e)\, BIC(M_2) = \ln P(D\,|\,\underline{\hat{\theta}}_{MLE}, M_2) - \frac{dof(M_2)}{2} \ln N = -9.5607 - \ln 7 = -11.5066$$

$$BIC(M_4) = \ln P(D\,|\,\underline{\hat{\theta}}_{MLE}, M_4) - \frac{dof(M_4)}{2} \ln N = -9.4625 - \frac{3}{2} \ln 7 = -12.1814$$

$BIC(M_2) > BIC(M_4) \Rightarrow BIC$ will also pick Model 2.

Problem 6

$$\frac{p(\underline{x} \mid z=1)}{p(\underline{x} \mid z=2)} \ge \frac{(\lambda_{12}-\lambda_{22})P(z=2)}{(\lambda_{21}-\lambda_{11})P(z=1)} \Rightarrow \hat{z}=1; \text{otherwise } \hat{z}=2$$

$$\Rightarrow \ln p(x \mid z=1) - \ln p(\underline{x} \mid z=2) \ge \ln\left[\frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}\right] + \ln P(z=2) - \ln P(z=1) \Rightarrow \textit{class } 1; \textit{else class } 2$$

$$\Rightarrow -\frac{1}{2}\ln(0.5) - \frac{1}{2}\frac{(x-2)^2}{0.5} + \frac{1}{2}\ln(0.2) + \frac{1}{2}\frac{(x-1.5)^2}{0.2} \ge \ln\underbrace{\left[\frac{\lambda_{12}-\lambda_{22}}{\lambda_{21}-\lambda_{11}}\right]}_{t} + \ln\frac{1}{3} - \ln\frac{2}{3} \Rightarrow \textit{class } 1; \textit{else class } 2$$

$$\Rightarrow 0.3466 - (x-2)^2 - 0.8047 + 2.5(x-1.5)^2 \ge \ln t - 0.6931 \Rightarrow \textit{class } 1; \textit{else class } 2$$

$$\Rightarrow 1.5x^2 - 3.5x + 1.7975 - \ln t \ge 0 \Rightarrow \textit{class } 1; \textit{else class } 2$$

$$t = 1 \Rightarrow \textit{class } 1 \ \textit{for } x \le 0.7632 \ \& \ x \ge 1.5701; \textit{class } 2 \ \textit{for } x \in (0.7632, 1.5701)$$